10. Circles

Exercise 10.1

1. Question

Fill in the blanks:

(ii) A circle may have parallel tangents.

(iii) A tangent to a circle intersects it in points(s).

(iv) A line intersecting a circle in two points is called a

(v) The angle between tangent at a point on a circle and the radius through the point is

Answer

(i) The tangent at any point of a circle is perpendicular to the radius through the point of contact.

- (ii) A circle may have two parallel tangents
- (iii) A tangent to a circle intersects it in one point.
- (iv) Secant is a line intersecting a circle in two points
- (v) The angle between tangent at a point on a circle and the radius through the point is 90°

2. Question

How many tangents can a circle have?

Answer

A circle can have infinite tangents.

3. Question

O is the centre of a circle of radius 8 cm. The tangent at a point A on the circle cuts a line through O at B such that AB = 15 cm. Find OB.

Answer

 $OB^2 = 8^2 + 15^2$ $OB = \sqrt{64 + 225}$ OB = 17 cm

4. Question

If the tangent at a point P to a circle with centre O cuts a line through O at Q such that PQ = 24 cm and OQ = 25 cm. Find the radius of the circle.

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Answer

since QT is a tangent to the circle at T and OT is radius,

Therefore OT perpendicular QT

It is given that OQ=25 cm and QT= 24 cm

By Pythagoras theorem we have



$$OB^{2} = OA^{2} + AB^{2}$$

 $OQ^{2} = QT^{2} + OT^{2}$
 $OT^{2} = 25^{2} - 24^{2}$
 $= 49 \times 1 = 49$
 $OT = 7$

Exercise 10.2

1. Question

If PT is a tangent at T to a circle whose centre is O and OP = 17 cm, OT = 8 cm, find the length of the tangent segment PT.

Answer



Given that O is the center of the circle and OP = 17 cm and the radius of the circle OT = 8 cm.

We need to find the length of the segment PT.

The line PT is the tangent line to the circle at the point T, the line through the centre is perpendicular to PT.

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OP^{2} = OT^{2} + PT^{2}

17^{2} = 8^{2} + PT^{2}

PT^{2} = 289 - 64

= 225

PT^{2} = \sqrt{225} = 15
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2. Question

Find the length of a tangent drawn to a circle with radius 5 cm, from a point 13 cm from the centre of the circle.

Answer



Given: PQ is a tangent to the circle intersect at OP=13cm and OQ=5 cm

Proof: In right triangle OQP

 $PQ=\sqrt{OP^2-OQ^2}=\sqrt{169-25}=12cm$

Therefore the length of the tan gent from the point is 12cm

3. Question

A point P is 26 cm away from the centre O of a circle and the length PT of the tangent drawn from P to the circle is 10 cm. Find the radius of the circle.

Answer

Let PQ be a tangent to the circle from point P and OQ be the radius at the point of contact.

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\therefore \angle OQP = 90^{\circ}
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\Rightarrow OP^2 = OQ^2 + PQ^2
(\because using Pythagoras theorem.

\Rightarrow OQ^2 = OP^2 - PQ^2 = 26^2 - 10^2
= (26 + 10)(26 - 10) = 36 \times 16
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$$\Rightarrow OQ=6 \times 4=24$$

∴ Radius of the circle = 24 cm.



4. Question

If from any point on the common chord of two intersecting circles, tangents be drawn to the circles, prove that they are equal.

Answer



Let the two circle intersect at a point X and Y , XY is the common chord.





Suppose A is a point on their common chord and AM and AN be the tangent drawn from A to the circle

AM is the tangent and AXY is a secant.

 $AM^2 = AX \times AY$ (i)

AN is the tangent and AXY is the secant.

 $AN^2 = AX \times AY$ (i)

Therefore, from equations (i) and (ii), we get,

AM = AN.

5. Question

If the sides of a quadrilateral touch a circle, prove that the sum of a pair of opposite sides is equal to the sum of the other pair.

Answer

Given: the sides of a quadrilateral touch a circle

To prove: the sum of a pair of opposite sides is equal to the sum of the other pair.

Proof:



From the theoram which states that the lengths of the two tangents drawn from an external point to a circle are equalFrom points A the tangents drawn are AP and AS, AP = AS (1)From points B the tangents drawn are BP and BQ, BP = BQ (2)From points D the tangents drawn are DR and DS, DR = DS(3)From points C the tangents drawn are CR and CQ, CR = CQ (4)Add 1,2,3 and 4 to getAP+BP+DR+CR = AS+BQ+DS+CQ(AP+BP)+(DR+CR) = (AS+DS)+ (BQ+CQ)AB+ DC = AD + BC

Hence proved

6. Question

If AB, AC, PQ are tangents in Fig. 10.51 and AB = 5 cm, find the perimeter of $\triangle APQ$.











Given: AB and Ac are tangent to the circle with centre O

PQ is tangent to the circle at X which intersect AB and Ac in P and Q

To find : Perimeter of triangle APQ

Proof:

AB = AC QC = QX PB = PX AB = AC = 5cm $Perimeter of \Delta APQ = AQ + QP + AP$ = AC + AB = 5 + 5 = 10cm

7. Question

Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.

Answer

Three tangent AB,CD and BD of a circle such as AB and CD are two parallel tangent BD intercept an angle BOD at the centre.

To Prove: $\angle BOD = 90^{\circ}$







Construction: Join OQ and OR Pr oof: $OP \perp BD$ Inrtght angle $\triangle OQB$ and OPB. $\triangle s OQB \cong OPB$. Sin ceOQ = OP, $\angle Q = \angle P$ and OB is common So $\angle 1 = \angle 2$ Simillarly in $\triangle s$ ORD and OPD we have $\angle 3 = \angle 4$ $\angle BOD = \angle 1 + \angle 3$ $= \frac{1}{2}(\angle 1 + \angle 2 + \angle 3 + \angle 4) = \frac{1}{2}(180^\circ) = 90^\circ$

8. Question

In Fig. 10.52, PQ is tangent at a point R of the circle with centre O. If $\angle TRQ = 30^{\circ}$, find $m \angle PRS$.



Answer

Given that, $\angle TRQ = 30^{\circ}$ Sin ce, ST is a diameter and angle in a semi – circle is at rt. angle Therefore, $\angle SRT = 90^{\circ}$ Now, $\angle TRQ + \angle SRT + \angle PRS = 180^{\circ}$ $30^{\circ} + 90^{\circ} + \angle PRS = 180^{\circ}$ $\angle PRS = 60^{\circ}$ 9. Question



If PA and PB are tangents from an outside point P. such that PA = 10 cm and $\angle APB = 60^{\circ}$. Find the length of chord AB.

Answer



Given: PA and PB are tangent of a circle PA= 10 cm and angle APB= 60°

Let O be the center of the given circle and C be the point of intersection of OP and AB

In triangle PAC and triangle PBC

PA = PB (tangent from an external point are equal)

APC =BPC (tangent from an external point are equally inclined to the segment joining center to the point)

 $AB = 5 + 5 = 10\,cm$

10. Question

From an external point P, tangents PA are drawn to a circle with centre O. If CD is the tangent to the circle at a point E and PA = 14 cm, find the perimeter of $\triangle PCD$.

Answer







Given: PA and PB are tangent to the circle with centre O

CD is tangent to the circle at E which intersect PA and PB in C and D

To find : Perimeter of triangle PCD

Proof:

PA = PB CA = DE DB = DE PA = PB = 14cm $Perimeter of \Delta PCD = PC + CD + PD$ = PA + PB = 14 + 14 = 28cm

11. Question

In Fig. 10.53, ABC is a right triangle right-angled at B such that BC = 6 cm and AB = 8 cm. Find the radius of its incircle.



Answer

Let ABC be the right angled triangle such that angle $B=90^{\circ}$, BC=6cm, AB=8cm. Let O be the centre and r be the radius of the in circle.







AB, BC and CA are tangent to the circle at P,N and M

OP=ON=OM=r (radius of the circle)

$$Area of \triangle ABC = \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$$

$$By Pythagoras theorem,$$

$$CA^2 = AB^2 + BC^2$$

$$CA^2 = 8^2 + 6^2$$

$$CA = 10 \text{ cm}$$

$$Area of \triangle ABC = Area of \triangle OAB + Area of \triangle OBC + Area of \triangle OCA$$

$$24 = \frac{1}{2}r \times AB + \frac{1}{2}r \times BC + \frac{1}{2}r \times CA$$

$$r = \frac{2 \times 24}{AB + BC + CA}$$

$$r = \frac{48}{8 + 6 + 10} = \frac{48}{24} = 2 \text{ cm}$$

12. Question

From a point P, two tangents PA and PB are drawn to a circle with centre O. If OP = diameter of the circle, show that $\triangle APB$ is equilateral.

Answer



AP is the tangent to the circle,

According to the theorem which states that tangent to a circle is perpendicular to the





radius through the point of contact.

 $\Rightarrow OA \perp AP$

 $\angle OAP = 90^{\circ}$

Also $OB \perp BP$

 $\Rightarrow \angle OBP = 90^{\circ}$

In Δ OAP sin θ =perpendicular/hypotenuse

 $\sin \angle OPA = \frac{r}{2r} = \frac{1}{2}$

As

 $\sin 30^{\circ} = \frac{1}{2}$ $\Rightarrow \angle OPA = 30^{\circ}$

Similarly $\angle OPB = 30^{\circ}Now \angle APB = \angle OPA + \angle OPB$

$$= 30^{\circ} + 30^{\circ}$$

$$= 60^{\circ} \dots (1)$$

In Δ PAB,As PA and PB are drawn from external point P,By theorem which states that the lengths of the two tangents drawn from external point to a circle are equal

$$\Rightarrow$$
 PA=PBAlso \angle PAB = \angle PBA (2)

As $\angle PAB + \angle PBA + \angle APB = 180^{\circ}$ (sum of angles of triangle)

$$\angle PAB + \angle PBA = 180^{\circ} - \angle APB$$

 $\angle PAB + \angle PBA = 180^{\circ} - 60^{\circ}$

 $\Rightarrow 2 \angle PAB = 120^{\circ}$

 $\Rightarrow \angle PAB = 60^{\circ} \dots (3)$

From 1 and 2 and $3, \angle PAB = \angle PBA = \angle APB = 60^{\circ}$ Hence $\triangle PAB$ is an equilateral triangle.

13. Question

Two tangent segments PA and PB are drawn to a circle with centre O such that $\angle APB = 120^{\circ}$. Prove that OP = 2 AP.

Answer

Given: Two tangent segments PA and PB are drawn to a circle with centre O such that $\angle APB = 120^{\circ}$.

To prove: OP = 2 AP

Proof:Construct the figure according to the conditions given.







HereIn triangle OAP and OBP,PA = PB (Length of Tangents from external point are equal)

OA = OB (Radii of same circle)

OP = OP (common)

 Δ OAP $\sim \Delta~$ OBP (By SSS criterion)

 $\angle OPA = \angle OPB = 60^{\circ}.$

In Triangle OAP , $\angle OAP = 90^{\circ}$ (By theoram which states that tangent to a circle is perpendicular to the radius through the point of contact)We know in a right angle triangle

 ${\sin}\theta = \frac{perpendicular}{hypotenuse}$

 \Rightarrow sin 60° = AP / OP , i.e 1/2 = AP / OP

So,OP = 2 AP

Hence proved.

14. Question

If $\triangle ABC$ is isosceles with AB = AC and C (O, r) is the incircle of the $\triangle ABC$ touching BC at L, prove that L bisect BC.

Answer

Given: If $\triangle ABC$ is isosceles with AB = AC and C (O, r) is the incircle of the $\triangle ABC$ touching BC at L.

To prove: L bisect BC.

Proof: Construct the figure according to given condition.







AB = AC (given)From the theorem which states that the lengths of two tangents drawn from external point to a circle are equal. (1) As tangents AP and AQ are drawn from the external point A.AP = AQ Also,AB = AC \Rightarrow AP + PB = AQ + QC \Rightarrow AP + PB = AP + QC \Rightarrow PB = QCFrom (1) as tangents BP and BL are drawn from external point B,And tangents CQ and CL are drawn from external point C. \Rightarrow BP = BL (3) CQ = CL (4)As we have proved PB = QCFrom 3 and 4BL = CL \Rightarrow L bisects BC.Hence proved.

15. Question

In Fig. 10.54, a circle touches all the four sides of a quadrilateral ABCD with AB = 6 cm, BC = 7 cm and CD = 4 cm. Find AD.



Answer







Here, AP = ASLet, AP = AS = XSimillarly, BP = BQCQ = CRRD = DSSin ce, AP = X $\Rightarrow BP = AB - AP = 6 - X$ Now, BP = BQ = 6 - XCQ = BC - BQ = 7 - (6 - X)= 1 + Xnow, CQ = CR = 1 + XRD = CD - CR = 4 - (1 + X)= 3 - XRD = DS = 3 - XAD = AS + SDX + 3 - X = 3AD = 3cmHere. AP = ASLet, AP = AS = XSimillarly, BP = BQCQ = CRRD = DSSin ce, AP = X $\Rightarrow BP = AB - AP = 6 - X$ Now, BP = BQ = 6 - XCQ = BC - BQ = 7 - (6 - X)= 1 + Xnow, CQ = CR = 1 + XRD = CD - CR = 4 - (1 + X)= 3 - XRD = DS = 3 - XAD = AS + SDX + 3 - X = 3AD = 3cm

16. Question

Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.





Answer



Draw a circle with centre O, draw a tangent PR touching circle at P.Draw QP perpendicular to RP at a point P, QP lies in the circle.Now, \angle OPR = 90°Also, \angle QPR = 90° Therefore, \angle OPR = \angle QPRThis is possible only when O lies on QP.Hence, it is proved that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

17. Question

In fig. 10.55, O is the centre of the circle and BCD is tangent to it at C. Prove that $\angle BAC + \angle ACD = 90^{\circ}$.



Answer



Given: In the above figure, O is the centre of the circle and BCD is tangent to it at C.To prove: \angle BAC + \angle ACD = 90°Proof:

In ∆OAC

OA = OC [radii of same circle]

 $\Rightarrow \angle OCA = \angle OAC$ [angles opposite to equal sides are equal]

 $\Rightarrow \angle OCA = \angle BAC$ [1]

Also,

 $OC \perp BD$ [Tangent at any point on a circle is perpendicular to the radius through point of contact]

⇒ ∠OCD = 90°

 $\Rightarrow \angle OCA + \angle ACD = 90^{\circ}$



 $\Rightarrow \angle BAC + \angle ACD = 90^{\circ}$ [From 1]

Hence Proved

18. Question

Two circles touch externally at a point P. From a point T on the tangent at P, tangents TQ and TR are drawn to the circles with points of contact Q and R respectively. Prove that TQ = TR



Answer

Let us label two circles as 'a' and 'b'



As TQ and TP are tangents to circle a,And TP and TR are tangents to circle b.By theorem which states that the lengths of the two tangents drawn from external point to a circle are equal.

TQ=TP ...(1)TP=TR ...(2)From 1 and 2,TQ=TRHence proved

19. Question

In Fig 10.57, a circle is inscribed in a quadrilateral ABCD in which $\angle B = 90^\circ$. If AD = 23 cm, AB = 29 cm and DS = 5 cm, find the radius r of the circle.

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Given: ABCD is a quadrilateral in which

 $\angle B = 90^{\circ}$ AD = 23 cm, DS = 5 cm and AB = 29 cm

Let the radius of the incircle be r cm.

AD = DS = 5cm (tangent from an external point)

Since AD=23 cm

So,

AR + RD + AD

AR + 5 = 23 cm

AR = 18 cm -----(i)

and AQ = AR

since AR = 18 cm

So, AQ + QB = AB

Now OP and OQ are radius of the circle. So from tangent P and Q

 $\angle OPB = \angle OQB = 90^{\circ}$ OPBQ is a square OP = QBRAdius of the circle = 11cm

20. Question





In Fig. 10.58, there are two concentric circles with centre O of radii 5 cm and 3 cm. from an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP.





Answer



PA and PB are the tangent drawn from the external

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point P to outer and inner circle respectively
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\angle OAP = 2 \ \angle OBP = 90^{\circ}
Given OA = 5 \ cm, OB = 3 \ cm and AP = 12 \ cm

In \ \triangle OAP

OP^2 = (12 \ cm)^2 + (5 \ cm)^2 = 169 \ cm^2

OP = 13 \ cm

IN \ \triangle OBP,

PB^2 = OP^2 - OB^2

PB^2 = (13 \ cm)^2 - (3 \ cm)^2 = 160 \ cm^2

PB = 4 \sqrt{10} \ cm

Thus the length of PB = 4 \sqrt{10} \ cm
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21. Question

In Fig. 10.59, AB is a cord of length 16 cm of a circle of radius 10 cm. The tangents at A and B intersect at a point P. Find the length of PA.



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Answer



OA=10 cm

As we know Perpendicular from centre to the chord bisects the chord.

So AM=MB=8cm

Using Pythagoras theorem in triangle AOM

$$OM = \sqrt{10^2 - 8^2} = 6 \, cm$$
$$\tan \angle AOM = \frac{8}{6} = \frac{4}{3}$$
$$Nowin \triangle OAP$$
$$\tan \angle AOM = \frac{PA}{OA}$$
$$PA = \frac{40}{3}$$

22. Question

In Fig. 10.60, PA and PB are tangents from an external point P to a circle with centre O. LN touches the circle at A. Prove that PL + LM = PN + MN.



Answer

PA = PB - - - - (i)As tangent drawn from external points to a circle are equal in length PL + AL = PN + BN - - - - - (ii)PLA and PNB are the two tengent which are equal AL = ML and BN = MN - - - - - (iii)From(ii) and (iii) PL = ML**23. Question**





In Fig. 10.61, BDC is a tangent to the given circle at point D such that BD = 30 cm and CD = 7 cm. The other tangents BE and CF are drawn respectively from B and C to the circle and meet when produced at A making BAC a right angle triangle. Calculate (i) AF (ii) radius of the circle.

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Answer

Given : AB, BC and AC are tangents to the circle at E, D and F.

BD = 30 cm and DC = 7 cm and \angle BAC = 90°

Recall that tangents drawn from an exterior point to a circle are equal in length

Hence BE = BD = 30 cmAlso FC = DC = 7 cmLet $AE = AF = x \rightarrow (1)$ Then AB = BE + AE = (30 + x)AC = AF + FC = (7 + x)BC = BD + DC = 30 + 7 = 37 cmConsider right Δ ABC, by Pythagoras theorem we have $BC^2 = AB^2 + AC^2$ $\Rightarrow (37)^2 = (30 + x)^2 + (7 + x)^2$ $\Rightarrow 1369 = 900 + 60x + x^2 + 49 + 14x + x^2$ $\Rightarrow 2x^2 + 74x + 949 - 1369 = 0$ $\Rightarrow 2x^2 + 74x - 420 = 0$ $\Rightarrow x^{2} + 37x - 210 = 0$ $\Rightarrow x^{2} + 42x - 5x - 210 = 0$ $\Rightarrow x (x + 42) - 5 (x + 42) = 0$ $\Rightarrow (x - 5) (x + 42) = 0$ \Rightarrow (x - 5) = 0 or (x + 42) = 0

 \Rightarrow x = 5 or x = - 42

 \Rightarrow x = 5 [Since x cannot be negative]

∴ AF = 5 cm [From (1)]

Therefore AB = 30 + x = 30 + 5 = 35 cm

AC = 7 + x = 7 + 5 = 12 cm

Let 'O' be the centre of the circle and 'r' the radius of the circle.

Join point O, F; points O, D and points O, E.

From the figure,

Area of ($\triangle ABC$) = Area ($\triangle AOB$) + Area ($\triangle BOC$) + Area ($\triangle AOC$)

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∴ r = 5
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Thus the radius of the circle is 5 cm

24. Question

In Fig. 10.62, $PO \perp OQ$. The tangents to the circle at P and Q intersect at a point T. Prove that PQ and OT are right bisectors of each other.







To prove: PQ and OT are the right bisectors.

Proof: To prove PQ and OT are the right bisectors,

We need to prove $\angle PRT = \angle TRQ = \angle QRO = \angle ORP = 90^{\circ}$

As it is given that $PO \perp OQ$,

$$\Rightarrow \angle POQ = 90^{\circ}$$



In Δ POT and $~\Delta$ OQT

OP=OQ (Radius) \angle OPT = \angle OQT = 90° (Tangent to a circle at a point is perpendicular to the radius through the point of contact)OT=OT (common)

∴ Δ POT \cong Δ OQTThus PT=OQ (BY C.P.C.T) (1) Now in Δ PRT and Δ ORQ ∠TPR = ∠OQR (alternate angles)∠PTO = ∠TOQ (alternate angles)PT=OQ (from (1)). Δ PRT \cong Δ ORQThus TQ = OP (By C.P.C.T 0Hence PT=TQ=OQ=OPThus it is a square, \Rightarrow The diagnols bisect at 90°.

Hence proved

25. Question

In Fig. 10.63, two tangents AB and AC are drawn to a circle with centre O such that $\angle BAC = 120^{\circ}$. Prove that OA = 2AB.



Answer



It can be clearly show that OA bisects angle CAB,

$$\angle OBC = \angle OBD = 60^{\circ}$$

$$In \, \triangle OAB$$

$$\angle OAB = 60^{\circ}, \, \angle OBA = 90^{\circ}$$

$$\angle BOA + \angle OAB + \angle OBA = 180^{\circ}$$

$$\angle BOA = 180^{\circ} - 150^{\circ} = 30^{\circ}$$

$$\sin (\angle BOA) = \frac{AB}{AO}$$

$$\sin 30^{\circ} = \frac{AB}{AO}$$

$$\frac{1}{2} = \frac{AB}{AO}$$

$$AO = 2AB$$





26. Question

In Fig. 10.64, BC is a tangent to the circle O. OE bisects AP. Prove that $\triangle AEO \sim \triangle ABC$.



Answer



Triangle AOP is an isosceles triangle because OA=OP as they are the radius of the circle. We know that radius of the circle is always perpendicular to the tangent at the point of contact.

Here OB is the radius and BC is the tangent and B is the point of contact, Therefore

 $\angle ABC = 90^{\circ}$

Also from the property of isoceles triangle we have found that

 $\angle OEA = 90^{\circ}$ Therefore, $\angle ABC = \angle OEA$ $\angle Ais common angle to both triangle$ Therefore, from AA postulates of similar triangle $\triangle AOE \sim \triangle ABC$

27. Question

The lengths of three consecutive sides of a quadrilateral circumscribing a circle are 4 cm, 5 cm and 7 cm respectively. Determine the length of the fourth side.

Answer







Given AB = 7, BC = 5, CD = 4Length of the tangent drawn from an external point to the circle are equal Therefore AB + CD = BC + AD7 + 4 = 5 + AD

AD = 11 - 5 = 6

28. Question

In Fig. 10.65, common tangents PQ and RS to two circles intersect at A. Prove that PQ = RS.



Answer



Given: PQ and RS are the two common tangent to the two circle

To Proof: A is the point of intersection of PQ and RS

We know that , length of two tangent drawn from an exterior point to acirclr are equal.

Therefore

PA = RA------(i)





QA = SA ----- (ii)

Adding two equations we get

PA + QA = RA + SA

PQ =RS (proved)

29. Question

Equal circles with centres O and O' touch each other at X. OO' produced to meet a circle with centre O', at A. AC is tangent to the circle whose centre is O. O' D is perpendicular to AC. Find the value of $\frac{DO'}{CO}$.



Answer

We know that $\angle ADO' = 90^{\circ}$ (since O'D is perpendicular to AC)

As we know radius is perpendicular to the tangent.

So, OC \perp AC

⇒ ∠ACO = 90°

In $\triangle ADO'$ and $\triangle ACO$,

 $\angle ADO' = \angle ACO \text{ (each 90°)}$

 $\angle DAO = \angle CAO$ (common)

By AA criteria,

 $\Delta ADO' \sim \Delta ACO$

As we know corresponding sides of a triangle are in ratio. $\frac{AO'}{AO} = \frac{DO'}{CO}$

AO = AO' + O'X + OX

As radii of two circles are equal.

$$\Rightarrow AO = AO' + AO' + AO'$$

= 3 AO'

$$\frac{AO'}{AO} = \frac{AO'}{3AO} = \frac{1}{3} \quad \frac{DO'}{CO} = \frac{AO'}{3AO} = \frac{1}{3} \quad \Rightarrow \frac{DO'}{CO} = \frac{1}{3}$$



30. Question

In Fig. 10.67, OQ:PQ=3:4 and perimeter of $\triangle POQ = 60$ cm. Determine PQ, OR and OP.



Answer



Given that OQ: PQ=3:4

Let ratio coefficient =x, so

OQ=3x and PQ=4x

We know that a tangent to a circle is perpendicular to the radius at the point of tangency

So

 $\angle OQP = 90^{\circ}$

Then applying Pythagoras theorem in triangle POQ

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OP^{2} = OQ^{2} + PQ^{2}

OP^{2} = (3x)^{2} + (4x)^{2}

OP^{2} = 9x^{2} + 16x^{2}

OP^{2} = 25x^{2}

OP = 5x

Perimeter of a triangle POQ is = 60 cm, So

3x + 4x + 5x = 60

12x = 60

x = 5

So,

OQ = 3x = 15cm

PQ = 4x = 20cm

OP = 5x = 25cm

QR = 2(OQ) = 2 \times 15 = 30 cm
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31. Question

Two concentric circles are of diameters 30 cm and 18 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer



In the diagram AB is the chord touching the smaller circle. We have the right angled triangle OO'B

By Pythagoras theorem

$$O'B = \sqrt{(30)^2 + (18)^2} = 24 \, cm$$

Now since the chord of the larger circle which touches the smaller circle is bisected at the point of contact

We have

AB= 2 × 24= 48 cm

So ans is 18 cm.

32. Question

A triangle PQR is drawn to circumscribe a circle of radius 8 cm such that the segments QT and TR, into which QR is divided by the point of contact T, are of lengths 14 cm and 16 cm respectively. If area of $\triangle PQR$ is 336 cm², find the sides PQ and PR.

Answer

Let PQ and PR touch the circle at points S and U respectively. Join O with P, Q, R, S and U



We have OS = OT = OU = 6cm





QT = 12 cm and TR = 9cm QR= QT +TR = 12cm + 9cm = 21 cm Now QT = QS=12 cm (tangent from the same point) TR =RU=9cm Let PS =PU =x cm

Then PQ=PS+SQ- (12 + x)cm and PR = PU+RU= (9+x)cm

It is clear that

$$ar (\triangle OQR) + ar (\triangle OPR) + ar (\triangle OPQ) = ar (\triangle PQR)$$

$$\frac{1}{2} \times QR \times OT + \frac{1}{2} \times PR \times OU + \frac{1}{2} \times PQ \times OS = 189 \, cm^2$$

$$\frac{1}{2} \times (12 + x) \times 6 + \frac{1}{2} \times (9 + x) \times 6 + \frac{1}{2} \times 21 \times 6 = 189$$

$$\frac{1}{2} \times 6 (12 + x + 9 + x + 21) = 189$$

$$3(42 + 2x) = 189$$

$$42 + 2x = 63$$

$$x = \frac{21}{2} = 10.5$$
Thus PQ = (12 + 10.5) cm = 22.5 cm and PR = (9 + 10.5) cm = 19.5 cm

33. Question

In Fig. 10.68, a $\triangle ABC$ is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC are of lengths 8 cm and 6 cm respectively. Find the lengths of sides AB and AC, when area of $\triangle ABC$ is 84 cm².



Answer







Firstly consider that the given circle will touch the given circle will touch the sides AB and AC of the triangle at a point E and F respectively.

Let AF=x

Now in triangle ABC

CF = CD = 6cm

(Tangent drawn from an external point to a circle are equal. Here tangent is drawn from external point C)

BE = BD = 8cm (Tangent drawn from an external point to a circle are equal. Here tangent is drawn from external point B)

AE = AF = X

Now AB = AE + EB = x + 8

Also BC = BD+ DC = 8+6=14 and CA= CF+FA = 6+x

Now we get all side of the triangle and its area can be find by using hero's formula





Semi - perimeter =
$$s = \frac{28 + 2x}{2} = 14 + x$$

Area of $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$
= $\sqrt{(14+x)\{(14+x)-14\}\{(14+x)(6+x)\}\{(14+x)-(8+x)\}}$
= $4\sqrt{3}(14x+x^2)$
Ar of $\triangle OBC = \frac{1}{2} \times 4 \times 14 = 28$
Ar of $\triangle OCA = \frac{1}{2} \times 4 \times (6+x) = 12 + 2x$
Ar of $\triangle OAB = \frac{1}{2} \times 4 \times (8+x) = 16 + 2x$
Area of $\triangle ABC = Ar$ of $\triangle OBC + Ar$ of $\triangle OCA + Ar$ of $\triangle OAB$
 $4\sqrt{3}(14x+x^2) = 28 + 12 + 2x + 16 + 2x$
 $\sqrt{3}(14x+x^2) = 14 + x$
Squarring both side and solving we get
 $x(x+14) - 7(x+14) = 0$
or $x = -14$ and 7
 $x = -14$ is not possible
so $x = 7$
hence $AB = 7 + 8 = 15$ cm
 $CA = 6 + 7 = 13$ cm

34. Question

In Fig. 10.69, AB is a diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58^{\circ}$, find $\angle ATQ$.













35. Question

In Fig. 10.70, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that \angle RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find \angle RQS.



Answer

As we know that the tangents drawn from an external point to a circle are equal.

Therefore, PQ = PR

Also, from the figure, PQR is an isosceles triangle, because PQ = PR

Therefore, $\angle RQP = \angle QRP$ (Because the corresponding angles of the equal sides of the isosceles triangle are equal)And, from the angle sum property of a triangle,

 $\angle RQP + \angle QRP + \angle RPQ = 180^{\circ}$

 $\angle RQP + \angle RQP + \angle RPQ = 180^{\circ}$

 $2 \angle RQP + \angle RPQ = 180^{\circ}$

 $2 \angle RQP + 30^{\circ} = 180^{\circ}$





 $2 \angle RQP = 180^{\circ} - 30^{\circ}$ $2 \angle RQP = 150^{\circ}$ $\angle RQP = 150^{\circ}/2$ Therefore, $\angle RQP = 75^{\circ}$ SR || QP and QR is a transversal $\therefore \angle SRQ = \angle PQR$...[Alternate interior angle] $\therefore \angle SRQ = 75^{\circ}$ $\Rightarrow \angle ORP = 90^{\circ}$...[Tangent is Perpendicular to the radius through the point of contact] $\angle ORP = \angle ORQ + \angle QRP$ \Rightarrow 90° = \angle ORQ + 75° $\Rightarrow \angle ORQ = 15^{\circ}$ Similarly, $\angle RQO = 15^{\circ}$ In Δ QOR, $\angle QOR + \angle QRO + \angle OQR = 180^{\circ}$ $\Rightarrow \angle QOR + 15^{\circ} + 15^{\circ} = 180^{\circ}$ $\Rightarrow \angle QOR = 150^{\circ}$ $\Rightarrow \angle QSR = \angle QOR/2$ $\Rightarrow \angle QSR = 150^{\circ}/2 = 75^{\circ}$ In ΔRSQ, $\angle RSQ + \angle QRS + \angle RQS = 180^{\circ}$ \Rightarrow 75° + 75° + \angle RQS = 180° ∠ROS = 30°

CCE - Formative Assessment

1. Question

In Fig. 10.72, PA and PB are tangents to the circle drawn from an external point P. CD are a third tangent touching the circle at Q. If PB = 10 cm and CQ = 2 cm, what is the length PC?







Answer

Given:

PB = 10 cm

CQ = 2 cm

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Using the above property,

PA = PB = 10 cm (tangent from P)

And,

```
CA = CQ = 10 \text{ cm} \text{ (tangent from C)}
```

Now,

PC = PA - CA

= 10 cm - 2 cm

= 8 cm

Hence, PC = 8 cm

2. Question

What is the distance between two parallel tangents of a circle of radius 4 cm?

Answer

Given:

Radius of circle (say PO) = 4 cm







Let AB || CD be two tangents which meets the circle at P and Q respectively. And, O be the center of circle.

<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, we can say that distance between two parallel tangents of a circle is equal to its diameter.

Therefore,

 $PQ = 2 \times PO$

= 2 × 4 cm

= 8 cm

Hence, Distance between tangents = 8 cm

3. Question

The length of tangent from a point A at a distance of 5 cm from the center of the circle is 4 cm. What is the radius of the circle?

Answer

Given:

OA(say) = 5 cm

AB = 4 cm









<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, $\triangle AOB$ is right-angled at $\angle OBA$.

Therefore, by Pythagoras Theorem,

 $AB^{2} + OB^{2} = AO^{2}$ $\Rightarrow OB^{2} = AO^{2} - AB^{2}$ $\Rightarrow OB = \sqrt{(AO^{2} - AB^{2})}$ $\Rightarrow OB = \sqrt{(5^{2} - 4^{2})}$ $\Rightarrow OB = \sqrt{(25 - 16)}$ $\Rightarrow OB = \sqrt{9}$ $\Rightarrow OB = 3 \text{ cm}$ Hence, Radius = 3 cm

4. Question

Two tangents TP and TQ are drawn from an external point T to a circle with center O as shown in Fig. 10.73. If they are inclined to each other at an angle of 100°, then what is the value of \angle POQ?







Fig. 10.73

Answer

Given:

 $\angle QTP = 100^{\circ}$

<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> Sum of all angles of a quadrilateral = 360°.

By property 1,

 $\angle OPT = 90^{\circ} \text{ and } \angle OQT = 90^{\circ}$

And,

By property 2,

 $\angle QTP + \angle OPT + \angle OQT + \angle POQ = 360^{\circ}$

 $\Rightarrow \angle POQ = 360^{\circ} - \angle QTP + \angle OPT + \angle OQT$

 $\Rightarrow \angle POQ = 360^{\circ} - 100^{\circ} + 90^{\circ} + 90^{\circ}$

⇒ ∠POQ = 360° - 280°

⇒ ∠POQ = 80°

Hence, $\angle POQ = 80^{\circ}$

5. Question

What the distance between two parallel tangents to a circle of radius 5 cm?

Answer

Given:

Radius of circle (say PO) = 5 cm







Let AB \parallel CD be two tangents which meets the circle at P and Q respectively. And, O be the center of circle.

<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, we can say that distance between two parallel tangents of a circle is equal to its diameter.

Therefore,

 $PQ = 2 \times PO$

= 2 × 5 cm

= 10 cm

Hence, Distance between tangents = 10 cm

6. Question

In Q. No. 1, if PB = 10 cm, what is the perimeter of Δ PCD?



Answer

Given:

PB = 10 cm

CQ = 2 cm




<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Using the above property,

PA = PB = 10 cm (tangent from P)

DB = DQ = 10 cm (tangent from D)

And,

CA = CQ = 10 cm (tangent from C)

Now,

Perimeter of $\triangle PCD = PC + CD + DP$

= PC + CQ + QD + DP

= PC + CA + DB + PD [::CA = CQ and DB = DQ]

= PA + PB [: PA = PC + CA and PB = PD + BD]

= 10 cm + 10 cm

= 20 cm

Hence, Perimeter of $\triangle PCD = 20$ cm

7. Question

In Fig. 10.74, CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm and BC = 7 cm, then find the length of BR.



Fig. 10.74

Answer

Given:

CP = 11 cm

BC = 7 cm

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Using the above property,

CP = CQ = 11 cm (tangent from C)

BQ = BR (tangent from B)





And,

AP = AR (tangent from A) Now, BR = BQ = CQ - CB = 11 cm - 7 cm = 4 cm Hence, BR = 4 cm

8. Question

In Fig. 10.75, Δ ABC is circumscribing a circle. Find the length of BC.





Answer

Given:

AR = 4 cm

BR = 3 cm

AC = 11 cm

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

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Using the above property,

AR = AQ = 4 cm (tangent from A)
BR= BP (tangent from B)
And,
CP = CQ (tangent from C)
Also,
CQ = CA - AQ = 11 cm - 4 cm = 7 cm
Now,

BC = BP + PC = BR + CQ [:: BR = BP and CP = CQ = 7 cm] = 3 cm + 7 cm = 10 cm

Hence, BC = 10 cm

9. Question

In Fig. 10.76, CP and CQ are tangents from an external point C to a circle with centre O. AB is another tangent which touches the circle at R. If CP = 11 cm and BR = 4 cm, find the length of BC.



1 ig. 10.70

[Hint: We have, CP = 11 cm

... CP = CQ = CQ = 11 cm

Now, BR = BQ [Tangents drawn from B)

 \implies BQ = 4 cm

BC = CQ - BQ = (11 - 4)cm = 7 cm

Answer

Given:

BR = 4 cm

CP = 11 cm

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

Using the above property,

BR = BQ = 4 cm (tangent from B)

And,

CP = CQ = 11 cm (tangent from C)

Now,

BC = CQ - BQ





= 11 cm - 4 cm

=7 cm

Hence, BC = 7 cm

10. Question

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer

Given:

AO (say) = CO (say) = 5 cm

BO (say) = 3 cm





<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, $\triangle AOB$ is right-angled at $\angle OBA$ and $\triangle COB$ is right-angled at $\angle OBC$.

Therefore,

By Pythagoras Theorem in $\triangle AOB$,

$$AB^2 + OB^2 = AO^2$$

$$\Rightarrow$$
 AB ² = AO² - OB ²

$$\Rightarrow$$
 AB= $\sqrt{(AO^2 - OB^2)}$

$$\Rightarrow$$
 AB= $\sqrt{(5^2 - 3^2)}$

$$\Rightarrow$$
 AB= $\sqrt{(25 - 9)}$

 $\Rightarrow AB = \sqrt{16}$





⇒ AB= 4 cm

Similarly,

By Pythagoras Theorem in ΔCOB ,

 $AB^2 + OB^2 = CO^2$

 \Rightarrow CB ² = CO² - OB ²

 \Rightarrow CB= $\sqrt{(CO^2 - OB^2)}$

 \Rightarrow CB= $\sqrt{(5^2 - 3^2)}$

 \Rightarrow CB= $\sqrt{(25 - 9)}$

 \Rightarrow CB = $\sqrt{16}$

 \Rightarrow CB= 4 cm

Now,

AC = AB + BC

= 4 cm + 4 cm

```
= 8 cm
```

Hence, Length of chord = 8 cm

11. Question

In Fig. 10.77, PA and PB are tangents to the circle with centre O such that $\angle APB = 50^{\circ}$. Write the measure of $\angle OAB$





Answer

Given:

∠APB = 50°

<u>Property 1:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.





<u>Property 2:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 3:</u> Sum of all angles of a triangle = 180°. By property 1, AP = BP (tangent from P) Therefore, $\angle PAB = \angle PBA$ Now, By property 3 in ΔPAB , $\angle PAB + \angle PBA + \angle APB = 180^{\circ}$ $\Rightarrow \angle PAB + \angle PBA = 180^{\circ} - \angle APB$ $\Rightarrow \angle PAB + \angle PBA = 180^{\circ} - 50^{\circ}$ $\Rightarrow \angle PAB + \angle PBA = 130^{\circ}$ $\Rightarrow \angle PAB = \angle PBA = \frac{130^{\circ}}{2} = 65^{\circ}$ By property 2, ∠PAO = 90° Now, $\angle PAO = \angle PAB + \angle OAB$ $\Rightarrow \angle OAB = \angle PAO - \angle PAB$ $\Rightarrow \angle OAB = 90^{\circ} - 65^{\circ} = 25^{\circ}$ Hence, $\angle OAB = 25^{\circ}$

1. Question

A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q such that OQ = 12 cm. Length PQ is cm

```
A. 12 cm
```

- B. 13 cm
- C. 8.5 cm

D. √119 cm

Answer

Given:

OQ = 12 cm







<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔPOQ is right-angled at $\angle OPQ$.

Therefore,

By Pythagoras Theorem in ΔPOQ ,

 $OP^2 + PQ^2 = OQ^2$

- $\Rightarrow PQ^2 = OQ^2 OP^2$
- \Rightarrow PQ= $\sqrt{(OQ^2 OP^2)}$
- \Rightarrow PQ= $\sqrt{(12^2 5^2)}$
- \Rightarrow PQ= $\sqrt{(144 25)}$
- \Rightarrow PQ = $\sqrt{119}$ cm

Hence, PQ = $\sqrt{119}$ cm

2. Question

From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

A. 7 cm

- B. 12 cm
- C. 15 cm
- D. 24.5 cm

Answer

Given:

OQ = 25 cm





PQ = 24 cm



<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, ΔPOQ is right-angled at $\angle OPQ$.

Therefore,

By Pythagoras Theorem in ΔPOQ,

 $OP^{2} + PQ^{2} = OQ^{2}$

 $\Rightarrow OP^2 = OQ^2 - PQ^2$

$$\Rightarrow$$
 OP= $\sqrt{(OQ^2 - PQ^2)}$

- \Rightarrow OP= $\sqrt{(25^2 24^2)}$
- \Rightarrow OP= $\sqrt{625 576}$
- \Rightarrow OP = $\sqrt{49}$ cm
- \Rightarrow OP = 7 cm

Hence, OP = 7 cm

3. Question

The length of the tangent from a point A at a circle, of radius 3 cm, is 4 cm. The distance of A from the centre of the circle is

A. √7 cm

B. 7 cm

C. 5 cm

D. 25 cm

Answer





Given:

AB (say) = 4 cm

Radius (OB) = 3 cm



<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, $\triangle AOB$ is right-angled at $\angle ABO$.

Therefore,

By Pythagoras Theorem in ΔPOQ ,

 $OA^2 = OB^2 + BA^2$

 \Rightarrow OA= $\sqrt{(OB^2 + BA^2)}$

 \Rightarrow OA= $\sqrt{(3^2 + 4^2)}$

 \Rightarrow OA= $\sqrt{(9 + 16)}$

 \Rightarrow OA = $\sqrt{25}$ cm

 \Rightarrow OA = 5 cm

Hence, distance of A from center = 5 cm

4. Question

If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80° then \angle POA is equal to

A. 50°

B. 60°

C. 70°

D. 80°





Answer

Given:

∠APB = 80°



<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

Property 2: Sum of all angles of a quadrilateral = 360°.

<u>Property 3:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By property 1, $\angle PAO = 90^{\circ}$ $\angle PBO = 90^{\circ}$ By property 2, $\angle APB + \angle PAO + \angle PBO + \angle AOB = 360^{\circ}$ $\Rightarrow \angle AOB = 360^{\circ} - \angle APB + \angle PAO + \angle PBO$ $\Rightarrow \angle AOB = 360^{\circ} - (80^{\circ} + 90^{\circ} + 90^{\circ})$ $\Rightarrow \angle AOB = 360^{\circ} - 260^{\circ}$ $\Rightarrow \angle AOB = 100^{\circ}$ Now, in $\triangle POA$ and $\triangle POB$ $OA = OB [\because radius of circle]$ PA = PB [By property 3 (tangent from P)] $OP = OP [\because common]$ $\therefore By SSS congruency,$ $\triangle POA \cong \triangle POB$





Hence, by CPCTC $\angle POA = \angle POB$ Now, $\angle AOB = 100^{\circ}$ $\Rightarrow \angle POA + \angle POB = 100^{\circ} [\because \angle AOB = \angle POA + \angle POB]$ $\Rightarrow \angle POA + \angle POA = 100^{\circ} [\because \angle POA = \angle POB]$ $\Rightarrow 2\angle POA = 100^{\circ}$ $\Rightarrow \angle POA = \frac{100^{\circ}}{2}$

Hence, $\angle POA = 50^{\circ}$

5. Question

If TP and TQ are two tangents to a circle with centre O so that $\angle POQ = 110^{\circ}$, then, $\angle PTQ$ is equal to

A. 60°

- B. 70°
- C. 80°
- D. 90°

Answer

Given:

```
∠POQ = 110°
```



<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

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<u>Property 2:</u> Sum of all angles of a quadrilateral = 360°.



By property 1,

∠TPO = 90°

∠TQO = 90°

By property 2,

 $\angle POQ + \angle TPO + \angle TQO + \angle PTQ = 360^{\circ}$

 $\Rightarrow \angle PTQ = 360^{\circ} - \angle POQ + \angle TPO + \angle TQO$

 $\Rightarrow \angle PTQ = 360^{\circ} - (110^{\circ} + 90^{\circ} + 90^{\circ})$

⇒ ∠PTQ = 360° - 290°

 $\Rightarrow \angle PTQ = 70^{\circ}$

Hence, $\angle PTQ = 70^{\circ}$

6. Question

PQ is a tangent to a circle with centre 0 at the point P. If A Δ OPQ is an isosceles triangle, then \angle OQP is equal to

- A. 30°
- B. 45°
- C. 60°
- D. 90°

Answer



<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> Sum of all angles of a triangle = 180° .

By property 1, $\triangle POQ$ is right-angled at $\angle OPQ$ (i.e., $\angle OPQ = 90^{\circ}$).

: ΔPOQ is an isosceles triangle





 $\therefore \angle POQ = \angle OQP$ By property 2, $\angle POQ + \angle OQP + \angle QPO = 180^{\circ}$ $\Rightarrow \angle POQ + \angle OQP = 180^{\circ} - \angle QPO$ $\Rightarrow \angle POQ + \angle OQP = 180^{\circ} - 90^{\circ}$ $\Rightarrow \angle POQ + \angle OQP = 180^{\circ} - 90^{\circ}$ $\Rightarrow \angle POQ + \angle OQP = 90^{\circ}$ $\Rightarrow \angle OQP + \angle OQP = 90^{\circ} [\because \angle POQ = \angle OQP]$ $\Rightarrow 2\angle OQP = 90^{\circ}$

$$\Rightarrow \angle OQP = \frac{90^{-1}}{2} \Rightarrow \angle OQP = 45^{\circ}$$

Hence, $\angle OQP = 45^{\circ}$

7. Question

Two equal circles touch each other externally at C and AB is a common tangent to the circles. Then, $\angle ACB =$

- A. 60°
- B. 45°
- C. 30°
- D. 90°

Answer



<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> Sum of all angles of a straight line = 180°.





Property 3: Sum of all angles of a triangle = 180°.

By property 1, $\triangle OAB$ is right-angled at $\angle OAB$ (i.e., $\angle OAB = 90^{\circ}$) and $\triangle PBA$ is right-angled at $\angle PBA$ (i.e., $\angle PBA = 90^{\circ}$)

Clearly,

 $\angle b + \angle c = \angle OAB$

 $\Rightarrow \angle b + \angle c = 90^{\circ}$ $\Rightarrow \angle b = 90^{\circ} - \angle c$

Similarly,

 $\angle d + \angle e = \angle PBA$

 $\Rightarrow \angle d + \angle e = 90^{\circ}$

⇒ ∠e = 90° - ∠d

Now,

 $\angle a = \angle b = 90^{\circ} - \angle c [:: OA = OC (Radius)]$

And,

 $\angle e = \angle f = 90^{\circ} - \angle d [:: PB = PC (Radius)]$

By property 2,

 $\angle a + \angle f + \angle ACB = 180^{\circ}$

 $\Rightarrow \angle ACB = 180^{\circ} - \angle a - \angle f$

 $\Rightarrow \angle ACB = 180^{\circ} - (90^{\circ} - \angle c) - (90^{\circ} - \angle d)$

 $\Rightarrow \angle ACB = 180^{\circ} - 90^{\circ} + \angle c - 90^{\circ} + \angle d$

 $\Rightarrow \angle ACB = \angle c + \angle d$

Now, in $\triangle ACB$

```
By property 3,
```

 $\angle ACB + \angle c + \angle d = 180^{\circ}$

 $\Rightarrow \angle ACB + \angle ACB = 180^{\circ} [\because \angle ACB = \angle c + \angle d]$

 $\Rightarrow 2 \angle ACB = 180^{\circ}$

 $\Rightarrow \angle ACB = \frac{180^{\circ}}{2}$ $\Rightarrow \angle ACB = 90^{\circ}$ Hence, $\angle ACB = 90^{\circ}$

8. Question



ABC is a right angled triangle, right angled at B such that BC = 6 cm and AB = 8 cm. A circle with centre O is inscribed in $\triangle ABC$. The radius of the circle is

A. 1 cm

B. 2 cm

C. 3 cm

D. 4 cm

Answer

Given:

BC = 6 cm

AB = 8 cm



<u>Property 1:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

<u>Property 2:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

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<u>Property 3:</u> Sum of all angles of a quadrilateral = 360° .

By property 1,

AP = AQ (Tangent from A)

BP = BR (Tangent from B)

CR = CQ (Tangent from C)

 \because ABC is a right-angled triangle, \div by Pythagoras Theorem

 $AC^2 = AB^2 + BC^2$



 $\Rightarrow AC^2 = 8^2 + 6^2$ $\Rightarrow AC^2 = 64 + 36$ $\Rightarrow AC^2 = 100$ $\Rightarrow AC = \sqrt{100}$ \Rightarrow AC = 10 cm Clearly, AQ + QC = AC = 10 cm \Rightarrow AP + RC = 10 cm [: AQ = AP and QC = RC] Also, AB + BC = 8 cm + 6 cm = 14 cm \Rightarrow AP + PB + BR + RC = 14 cm [: AB = AP + PB and BC = BR + RC] \Rightarrow AP + RC + PB + BR = 14 cm \Rightarrow 10 cm + BR + BR = 14 cm [: AP + RC = 10 cm and PB = BR] \Rightarrow 10 cm + 2BR = 14 cm \Rightarrow 2BR = 14 cm - 10 cm = 4 cm \Rightarrow BR = $\frac{4}{2}$ cm \Rightarrow BR = 2 cm Now, $\angle BPO = 90^{\circ} [By property 3]$ $\angle BRO = 90^{\circ}$ [By property 3] $\angle PBM = 90^{\circ}$ [Given] Now by property 2, $\angle BPO + \angle BRO + \angle PBM + \angle ROP = 360^{\circ}$ $\Rightarrow \angle ROP = 360^{\circ} - \angle BPO + \angle BRO + \angle PBM$ $\Rightarrow \angle ROP = 360^{\circ} - (90^{\circ} + 90^{\circ} + 90^{\circ})$ ⇒ ∠ROP = 360° - 270° $\Rightarrow \angle ROP = 90^{\circ}$ Now, $\because \angle ROP = 90^{\circ}$ and BP = BR which are adjacent sides : Quadrilateral PBRO is a square



 \Rightarrow PO = BR = 2 cm

Hence, Radius = 2 cm

9. Question

PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that \angle POR = 120°, then \angle OPQ is

A. 60°

- B. 45°
- C. 30°
- D. 90°

Answer

Given:

 $\angle POR = 120^{\circ}$



<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> Sum of all angles of a straight line = 180°.

Property 3: Sum of all angles of a triangle = 180°.

By property 1,

 $\angle PQO = 90^{\circ}$

By property 2,

 $\angle POQ + \angle POR = 180^{\circ}$

 $\Rightarrow \angle POQ + 120^{\circ} = 180^{\circ}$

⇒ ∠POQ = 180° - 120°





10. Question

If four sides of a quadrilateral ABCD are tangential to a circle, then

A. AC + AD = BD + CDB. AB + CD = BC + ADC. AB + CD = AC + BCD. AC + AD = BC + DB

Answer



<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

AP = AS (tangent from A)

- BP = BQ (tangent from B)
- CR = CQ (tangent from C)
- DR = DS (tangent from D)



Now we add above 4 equations,

AP + BP + CR + DR = AS + BQ + CQ + DS $\Rightarrow AB + CD = AD + BC$ [:: AP + BP = AB CR + DR = CD AS + DS = ADBQ + CQ = BC]

Hence, the right option is AB + CD = AD + BC

11. Question

The length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm is

A. √7 cm

B. 2√7 cm

- C. 10 cm
- D. 5 cm

Answer

Given:

OA = 6 cm

OB = 8 cm



<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

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By above property, $\triangle AOB$ is right-angled at $\angle OAB$ (i.e., $\angle OAB = 90^{\circ}$).

Therefore by Pythagoras theorem,

 $OA^{2} + AB^{2} = OB^{2}$ $\Rightarrow AB^{2} = OB^{2} - OA^{2}$ $\Rightarrow AB^{2} = 8^{2} - 6^{2}$ $\Rightarrow AB^{2} = 64 - 36$ $\Rightarrow AB^{2} = 28$ $\Rightarrow AB = \sqrt{28}$ $\Rightarrow AB = 2\sqrt{7}$

Hence, length of tangent is $2\sqrt{7}$ cm.

12. Question

AB and CD are two common tangents to circles which touch each other at C. If D lies on AB such that CD = 4 cm, then AB is equal to

- A. 4 cm
- B. 6 cm
- C. 8 cm
- D. 12 cm

Answer

Given:

AB and CD are two common tangents to circles which touch each other at C. If D lies on AB such that CD = 4 cm

To find: length of AB

Solution:



<u>Property</u>: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.





Since c is the external point to the circles and two common tangents touch each other at c.

By the above property,

AD = BD = CD = 4 cm (tangent from D)

Now clearly,

AB = AD + BD

- $\Rightarrow AB = AD + BD$
- \Rightarrow AB = 4 cm + 4 cm
- $\Rightarrow AB = 8 cm$

Hence, AB = 8 cm

13. Question

In Fig. 10.78, if AD, AE and BC are tangents to the circle at D, E and F respectively. Then,



Fig. 10.78

- A. AD = AB + BC + CA
- B. 2AD = AB + BC + CA
- C. 3AD = AB + BC + CA
- D. 4AD = AB + BC + CA

Answer

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

AE = AD (tangent from A)

AB = AC (tangent from A)

CD = CF (tangent from C)

BF = BE (tangent from B)

Now adding the above equations,

AB + BC + CA = AB + BF + FC + CA





 $\Rightarrow AB + BC + CA = AB + BE + CD + CA$ $\Rightarrow AB + BC + CA = AE + AD [:: AE = AB + BE and AD = AC + CD]$ $\Rightarrow AB + BC + CA = AD + AD [:: AD = AE]$ $\Rightarrow AB + BC + CA = 2AD$

Hence, 2AD = AB + BC + CA

14. Question

In Fig. 10.79, RQ is a tangent to the circle with centre O. If SQ = 6 cm and QR = 4 cm, then OR =



- A. 8 cm
- B. 3 cm
- C. 2.5 cm
- D. 5 cm

Answer

Given:

SQ = 6 cm

QR = 4 cm

<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, $\triangle ROQ$ is right-angled at $\angle OQR$ (i.e., $\angle OQR = 90^{\circ}$).

Diameter QS = 6 cm

Radius = $\frac{\text{Diameter}}{2}$ ⇒ Radius = $\frac{6 \text{ cm}}{2}$ ⇒Radius (OQ) = 3 cm

Now by Pythagoras theorem,



 $OR^{2} = OQ^{2} + QR^{2}$ $\Rightarrow OR^{2} = 3^{2} + 4^{2}$ $\Rightarrow OR^{2} = 9 + 16$ $\Rightarrow OR^{2} = 25$ $\Rightarrow OR = \sqrt{25}$ $\Rightarrow OR = 5 \text{ cm}$

Hence, OR= 5 cm.

15. Question

In Fig. 10.80, the perimeter of ΔABC is



AQ = 4 cm

BR = 6 cm

PC = 5 cm

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

AR = AQ = 4 cm (tangent from A)

BR = BP = 6 cm (tangent from B)

```
CP = CQ = 5 cm (tangent from C)
```

Now,





```
Perimeter of \triangle ABC = AB + BC + CA

\Rightarrow Perimeter of \triangle ABC = AR + RB + BP + PC + CQ + QA

[\because AB = AR + RB

BC = BP + PC

CA = CQ + QA]

\Rightarrow Perimeter of \triangle ABC = 4 \text{ cm} + 6 \text{ cm} + 6 \text{ cm} + 5 \text{ cm} + 5 \text{ cm} + 4 \text{ cm}

\Rightarrow Perimeter of \triangle ABC = 30 \text{ cm}
```

Hence, Perimeter of $\triangle ABC = 30$ cm

16. Question

In Fig. 10.81, AP is a tangent to the circle with centre O such that OP = 4 cm and \angle OPA = 30°. Then, AP =



A. 2√2 cm

B. 2 cm

C. 2√3cm

D. 3√2 cm

Answer

Given:

OP = 4 cm

∠OPA = 30°







<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, $\triangle POA$ is right-angled at $\angle OAP$ (i.e., $\angle OAP = 90^{\circ}$).

Now we know that,

$$\cos \Theta = \frac{\text{Base}}{\text{Hypotnuse}}$$

Therefore,

$$\cos \angle P = \frac{AP}{OP}$$
$$\Rightarrow \cos 30^\circ = \frac{AP}{4}$$
$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AP}{4}$$
$$\Rightarrow AP = \frac{4 \times \sqrt{3}}{2}$$

 $\Rightarrow AP = 2\sqrt{3} cm$

Hence, AP = $2\sqrt{3}$ cm

17. Question

AP and PQ are tangents drawn from a point A to a circle with centre O and radius 9 cm. If OA = 15 cm, then AP + AQ =

A. 12 cm

B. 18 cm

C. 24cm

D. 36 cm

Answer

Given:

Radius = 9 cm

OA = 15 cm





<u>Property 1:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

AP = AQ (tangent from A)

<u>Property 2:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, $\triangle POA$ is right-angled at $\angle OAP$ (i.e., $\angle OPA = 90^{\circ}$).

Therefore by Pythagoras theorem,

$$AP^{2} + PO^{2} = AO^{2}$$

$$\Rightarrow AP^{2} = AO^{2} - PO^{2}$$

$$\Rightarrow AP^{2} = 15^{2} - 9^{2}$$

$$\Rightarrow AP^{2} = 225 - 81$$

$$\Rightarrow AP^{2} = 144$$

$$\Rightarrow AP = \sqrt{144}$$

$$\Rightarrow AP = 12$$

$$AP + AQ = 12 \text{ cm} + 12 \text{ cm} = 24 \text{ cm}$$

Hence, $AP + AQ = 24 \text{ cm}$
18. Ouestion

At one end of a diameter PQ of a circle of radius 5 cm, tangent XPY is drawn to the circle. The length of chord AB parallel to XY and at a distance of 8 cm from P is

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A. 5 cm

B. 6 cm

C. 7 cm

D. 8 cm

Answer

Given:

Radius = OP = 5 cm

Distance of AB and XY = 8 cm



 \therefore Distance of AB and XY = 8 cm

And AB is parallel to XY

 \therefore PR = 8 cm

Join OB

Now,

OB = OP = 5 cm [radius]

Also,

OR = PR - PO

 \Rightarrow OR = 8 cm - 5 cm

 \Rightarrow OR = 3 cm

 \therefore By Pythagoras theorem in $\Delta ORB,$

 $OB^2 = OR^2 + RB^2$

 $\Rightarrow 5^2 = 3^2 + RB^2$

 $\Rightarrow RB^{2} = 5^{2} - 3^{2}$ $\Rightarrow RB^{2} = 25 - 9$ $\Rightarrow RB^{2} = 16$ $\Rightarrow RB = 4$ Now, AB = AR + RB $\Rightarrow AB = 2RB$ $\Rightarrow AB = 2 \times 4$ $\Rightarrow AB = 8 \text{ cm}$

Hence, Length of chord = 8 cm

19. Question

If PT is tangent drawn from a point P to a circle touching it at T and O is the centre of the circle, then $\angle OPT + \angle POT =$

- A. 30°
- B. 60°
- C. 90°
- D. 180°

Answer



<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

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<u>Property 2:</u> Sum of all angles of a triangle = 180°

By property 1, \triangle PTO is right-angled at \angle OTP (i.e., \angle OTP = 90°).



By property 2, $\angle OTP + \angle POT + \angle TPO = 180^{\circ}$ $\Rightarrow 90^{\circ} + \angle POT + \angle TPO = 180^{\circ}$ $\Rightarrow \angle POT + \angle TPO = 180^{\circ} - 90^{\circ}$ $\Rightarrow \angle POT + \angle TPO = 90^{\circ}$

Hence, $\angle POT + \angle TPO = 90^{\circ}$

20. Question

In the adjacent figure, if AB = 12 cm, BC = 8 cm and AC = 10 cm, then AD =



- A. 5 cm
- B. 4 cm

C. 6 cm

D. 7 cm

Answer

Given:

AB = 12 cm

BC = 8 cm

AC = 10 cm

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

AD = AF (tangent from A)

BD = BE (tangent from B)

CF = CE (tangent from C)

Clearly,

AB = AD + DB = 12 cm





BC = BE + EC = 8 cm
AC = AF + FC = 10 cm
Now,
AB - BC = 12 cm - 8 cm

$$\Rightarrow$$
 (AD + DB) - (BE + EC) = 12 cm - 8 cm
 \Rightarrow AD + DB - BE - EC = 12 cm - 8 cm
 \Rightarrow AD + BE - BE - CF = 12 cm - 8 cm [\because DB = BE and CF = CE]
 \Rightarrow AD - CF = 12 cm - 8 cm
 \Rightarrow AD - (10 cm - AF) = 12 cm - 8 cm [\because AF + FC = 10 cm \Rightarrow FC = 10 cm - AF]
 \Rightarrow AD - (10 cm - AF) = 4 cm
 \Rightarrow AD - 10 cm + AF = 4 cm
 \Rightarrow AD + AD = 4 cm + 10 cm [\because AD = AF]
 \Rightarrow 2AD = 14 cm
 \Rightarrow AD = $\frac{14 \text{ cm}}{2}$
 \Rightarrow AD = 7 cm
Hence, AD = 7 cm

21. Question

In Fig. 10.83, if AP = PB, then



D. AB = BC

Answer





Given:

AP = PB

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

AP = AQ (tangent from A)

BR = BP (tangent from B)

CQ = CR (tangent from C)

Clearly,

AP = BP = BR

AQ = AP = BR

Now,

AQ + QC = BR + RC

 \Rightarrow AC = BC [::AC = AQ + QC and BC = BR + RC]

Hence, AC = BC

22. Question

In Fig. 10.84, if AP = 10 cm, then BP =



- A. $\sqrt{109}$ cm
- B. $\sqrt{127}$ cm
- c. √119 cm

D. √109 cm

Answer

Given:

AP = 10 cm





OA = 6 cm

OB = 3 cm

<u>Property</u> : The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, $\triangle PAO$ is right-angled at $\angle PAO$ (i.e., $\angle PAO = 90^{\circ}$) and $\triangle PBO$ is right-angled at $\angle PBO$ (i.e., $\angle PBO = 90^{\circ}$).

Therefore by Pythagoras theorem in ΔPAO ,

 $OP^2 = OA^2 + AP^2$

 $\Rightarrow OP^2 = 6^2 + 10^2$

 $\Rightarrow OP^2 = 36 + 100$

$$\Rightarrow$$
 OP= $\sqrt{136}$

Now by Pythagoras theorem in $\triangle PBO$,

 $OP^2 = OB^2 + BP^2$

 $BP^2 = OP^2 - OB^2$

 $\Rightarrow BP^2 = (\sqrt{136})^2 - 3^2$

 $\Rightarrow BP^2 = 136 - 9$

$$\Rightarrow$$
 BP= $\sqrt{127}$

Hence, BP= $\sqrt{127}$ cm

23. Question

In Fig. 10.85, if PR is tangent to the circle at P and Q is the centre of the circle, then $\angle POQ =$







Given:

 $\angle RPQ = 60^{\circ}$

<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> Sum of all angles of a triangle = 180°.

By property 1, $\triangle OPR$ is right-angled at $\angle OPR$ (i.e., $\angle OPR = 90^{\circ}$).

OP = OQ [:: radius of circle]

 $\therefore \angle OPQ = \angle OQP = 30^{\circ}$

Now by property 2,

 $\angle OPQ + \angle OQP + \angle POQ = 180^{\circ}$

 $\Rightarrow 30^{\circ} + 30^{\circ} + \angle POQ = 180^{\circ}$

 $\Rightarrow 60^{\circ} + \angle POQ = 180^{\circ}$

⇒ ∠POQ = 180° - 60°

 $\Rightarrow \angle POQ = 120^{\circ}$

Hence, $\Rightarrow \angle POQ = 120^{\circ}$

24. Question

In Fig. 10.86, if quadrilateral PQRS circumscribes a circle, then PD + QB =



- A. PQ
- B. QR
- C. PR
- D. PS

Answer

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,





```
PD = PA (tangent from P)

QB = QA (tangent from Q)

RC = RB (tangent from R)

SC = SD (tangent from S)

Now,

PD + QB = PA + QA

\Rightarrow PD + QB = PQ [\becausePQ = PA + QA]
```

Hence, PD + QB = PQ

25. Question

In Fig. 10.87, two equal circles touch each other at T, if QP = 4.5 cm, then QR =



A. 9 cm

B. 18 cm

C. 15 cm

D. 13.5 cm

Answer

Given:

QP = 4.5 cm

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

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By the above property,

PQ = PT = PR = 4.5 cm (tangent from P)

Now,

QR = PQ + PR

QR = PQ + PQ [::PQ = PR]

QR = 2PQ

 $QR = 2 \times 4.5 \text{ cm}$

QR = 9 cm

Hence, QR = 9 cm

26. Question

In Fig. 10.88, APB is a tangent to a circle with centre O at point P. If \angle QPB = 500, then the measure of \angle POQ is



A. 100°

B. 120°

C. 140°

D. 150°

Answer

Given:

∠QPB =50°

<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> Sum of all angles of a triangle = 180°.

By property 1, $\triangle OPB$ is right-angled at $\angle OPB$ (i.e., $\angle OPB = 90^{\circ}$).

 $\angle OPQ = \angle OPB - \angle QPB$

 $\Rightarrow \angle OPQ = 90^{\circ} - 50^{\circ} = 40^{\circ}$

And,

 $\angle OPQ = \angle OQP [:: OP = OQ (radius of circle)]$

Now by property 2,

 $\angle OPQ + \angle OQP + \angle POQ = 180^{\circ}$

 $\Rightarrow 40^{\circ} + 40^{\circ} + \angle POQ = 180^{\circ}$

 $\Rightarrow 80^{\circ} + \angle POQ = 180^{\circ}$

 $\Rightarrow \angle POQ = 180^{\circ} - 80^{\circ}$

 $\Rightarrow \angle POQ = 100^{\circ}$





Hence, $\Rightarrow \angle POQ = 100^{\circ}$

27. Question

In Fig. 10.89, if tangents PA and PB are drawn to a circle such that $\angle APS = 30^{\circ}$ and chord AC is drawn parallel to the tangent PB, then $\angle ABC =$



- A. 60°
- B. 90°
- C. 30°

D. None of these

Answer

Given:

∠APB = 30°

<u>Property 1:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

```
Property 2: Sum of all angles of a triangle = 180°
```

By property 1,

PA = PB (tangent from P)

And,

```
\angle PAB = \angle PBA [::PA = PB]
```

By property 2,

```
\angle PAB + \angle PBA + \angle APB = 180^{\circ}
```

```
\Rightarrow \angle PAB + \angle PBA + 30^{\circ} = 180^{\circ}
```

```
\Rightarrow \angle PAB + \angle PBA = 180^{\circ} - 30^{\circ}
```

```
\Rightarrow \angle PAB + \angle PBA = 150^{\circ}
```

```
\Rightarrow \angle \mathsf{PBA} + \angle \mathsf{PBA} = 150^{\circ} [::\angle \mathsf{PAB} = \angle \mathsf{PBA}]
```

 $\Rightarrow 2 \angle PBA = 150^{\circ}$


$$\Rightarrow \angle PBA = \frac{150^{\circ}}{2}$$

 $\Rightarrow \angle PBA = 75^{\circ}$

Now,

 $\angle PBA = \angle CAB = 75^{\circ}$ [Alternate angles]

 \angle PBA = \angle ACB = 75° [Alternate segment theorem]

Again by property 2,

 $\angle CAB + \angle ACB + \angle CBA = 180^{\circ}$

$$\Rightarrow 75^{\circ} + 75^{\circ} + \angle CBA = 180^{\circ}$$

 \Rightarrow 150° + ∠CBA = 180°

⇒ ∠CBA = 180° - 150°

 $\Rightarrow \angle CBA = 30^{\circ}$

Hence, $\angle CBA = 30^{\circ}$

28. Question

In Fig. 10.90, PR =



A. 20 cm

B. 26 cm

C. 24 cm

D. 28 cm

Answer

Given:

QP = 4 cm

OQ = 3 cm

SR = 12 cm



SO' = 5 cm

<u>Property:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By above property, $\triangle OPQ$ is right-angled at $\angle OQP$ (i.e., $\angle OQP = 90^{\circ}$) and $\triangle O'SR$ is right-angled at $\angle O'SR$ (i.e., $\angle O'SR = 90^{\circ}$).

By Pythagoras theorem in $\triangle OPQ$,

 $\mathsf{O}\mathsf{P}^2 = \mathsf{Q}\mathsf{P}^2 + \mathsf{O}\mathsf{Q}^2$

 $\Rightarrow OP^2 = 4^2 + 3^2$

 $\Rightarrow OP^2 = 16 + 9$

 $\Rightarrow OP^2 = 25$

 $\Rightarrow OP = \sqrt{25}$

 \Rightarrow OP = 5 cm

By Pythagoras theorem in $\Delta O'SR$,

 $O'R^2 = SR^2 + O'S^2$

- $\Rightarrow O'R^2 = 12^2 + 5^2$
- $\Rightarrow 0'R^2 = 144 + 25$
- $\Rightarrow O'R^2 = 169$
- \Rightarrow O'R = $\sqrt{169}$
- \Rightarrow O'R² = 13 cm

Now,

PR = PO + ON + NO' + O'R

 \Rightarrow PR = 5 cm + 3 cm + 5 cm + 13 cm

 \Rightarrow PR = 26 cm

Hence, PR = 26 cm

29. Question

Two circles of same radii r and centres O and O' touch each other at P as shown in Fig. 10.91. If 00' is produced to meet the circle C (O', r) at A and AT is a tangent to the circle C(O, r) such that $O'Q \perp AT$. Then AO: AO' =







- A. 3/2
- B. 2
- C. 3
- D. 1/4

Answer

Given:

AO' = r

O'P = r

PO = r

AO = AO' + O'P + PO

 \Rightarrow AO = r + r + r

 $\Rightarrow AO = 3r$

Now,

$$\frac{AO}{AO'} = \frac{3r}{r} = 3$$

Hence, AO: AO' = 3

30. Question

Two concentric circles of radii 3 cm and 5 cm are given. Then length of chord BC which touches the inner circle at P is equal to



A. 4 cm





B. 6 cm

C. 8 cm

D. 10 cm

Answer

Given:

OA = 5 cm

OQ = 3 cm

<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By property 1, $\triangle OAQ$ is right-angled at $\angle OQA$ (i.e., $\angle OQA = 90^{\circ}$).

By Pythagoras theorem in ΔOAQ ,

```
OA^2 = OA^2 + OO^2
\Rightarrow OA^2 = OA^2 - OO^2
\Rightarrow OA^2 = 5^2 - 3^2
\Rightarrow QA^2 = 25^2 - 9^2
\Rightarrow OA^2 = 16
\Rightarrow OA = \sqrt{16}
\Rightarrow QA = 4 cm
By property 2,
BQ = BP (tangent from B)
And,
AQ = BQ = 4 \text{ cm} [: Q \text{ is midpoint of AB}]
PB = PC = 4 \text{ cm} [:: P \text{ is midpoint of } BC]
Now,
BC = BP + PC
\Rightarrow BC = 4 cm + 4 cm
\Rightarrow BC = 8 cm
Hence, BC = 8 \text{ cm}
31. Question
```



In Fig. 10.93, there are two concentric circles with centre O. PR and PQS are tangents to the inner circle from point plying on the outer circle. If PR = 7.5 cm, then PS is equal to



Fig. 10.93

A. 10 cm

B. 12 cm

C. 15 cm

D. 18 cm

Answer

Given:

PR = 7.5 cm



<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By property 1, $\triangle OSQ$ is right-angled at $\angle OQS$ (i.e., $\angle OQS = 90^{\circ}$) and $\triangle OPQ$ is right-angled at $\angle OQP$ (i.e., $\angle OQP = 90^{\circ}$).

 $\therefore \text{ OQ} \perp \text{PS}$

- : PO = OS [radius of circle]
- \therefore \DeltaPOS is an isosceles triangle

Now,





 $\cdot\cdot$ ΔPOS is an isosceles triangle and OQ is perpendicular to its base

.. OQ bisects PS i.e., PQ = QS By property 2, PR = PQ = 7.5 cm (tangent from P) Now, PS = PQ + QS \Rightarrow PS = PQ + PQ [:: PQ = QS] \Rightarrow PS = 7.5 cm + 7.5 cm \Rightarrow PS = 15 cm

Hence, PS = 15 cm

32. Question

In Fig. 10.94, if AB = 8 cm and PE = 3 cm, then AE =





- A. 11 cm
- B. 7 cm
- C. 5 cm
- D. 3 cm

Answer

Given:

AB = 8 cm

PE = 3 cm

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By above property,

AB = AC = 8 cm (tangent from A)

PE = CE = 3 cm (tangent from E)





Now,

AE = AC − CE \Rightarrow AE = 8 cm − 3 cm \Rightarrow AE = 5 cm Hence, AE = 5 cm

33. Question

In Fig. 10.95, PQ and PR are tangents drawn from P to a circle with centre O. If $\angle OPQ = 35^{\circ}$, then



Answer

Given:

 $_{/}OPQ = 35^{\circ}$

<u>Property 1:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

<u>Property 2:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 3:</u> The sum of all angles of a triangle = 180°.

By property 1,

QP = QR (tangent from Q)

By property 2, $\triangle OPQ$ is right-angled at $\angle OQP$ (i.e., $\angle OQP = 90^{\circ}$) and $\triangle ORP$ is right-angled at $\angle ORP$ (i.e., $\angle ORP = 90^{\circ}$).

 $\therefore \ \mathsf{OQ} \perp \mathsf{QP}$

OR⊥ RP

Now,

 $\angle OQP = \angle ORP = 90^{\circ}$ [Property 1]





QP = QR [Property 2]OP = OP [Given] $\therefore \Delta OPQ \cong \Delta OPR By SAS$ Hence, $\angle OPQ = \angle OPR = 35^{\circ}$ By CPCTC i.e. ∠a = 35° By property 3, $\angle OQP + \angle OPQ + \angle QOP = 180^{\circ}$ \Rightarrow 90° + 35° + \angle QOP = 180° $\Rightarrow 125^{\circ} + \angle QOP = 180^{\circ}$ ⇒ ∠OOP = 180° - 125° ⇒ ∠QOP = 55° i.e. ∠b = 55° Hence, $\angle a = 35^{\circ}$ and $\angle b = 55^{\circ}$

34. Question

In Fig. 10.96, if TP and TQ are tangents drawn from an external point T to a circle with centre O such that $\angle TQP = 60^{\circ}$, then $\angle OPQ =$



A. 25°

B. 30°

C. 40°

D. 60°

Answer

Given:

 $_{\angle}TQP = 60^{\circ}$

Property 1: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.





<u>Property 2:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

By property 1,

TP = TQ (tangent from T)

 $\Rightarrow \angle TPQ = \angle TQP = 60^{\circ}$

By property 2, $\triangle OPT$ is right-angled at $\angle OPT$ (i.e., $\angle OPT = 90^{\circ}$) and $\triangle OQT$ is right-angled at $\angle OQT$ (i.e., $\angle OQT = 90^{\circ}$).

Now,

 $\angle OPQ = \angle OPT - \angle TPQ$

 $\Rightarrow \angle OPQ = 90^{\circ} - 60^{\circ}$

 $\Rightarrow \angle OPQ = 30^{\circ}$

Hence, ∠OPQ = 30°

35. Question

In Fig. 10.97, the sides AB, BC and CA of triangle ABC, touch a circle at P, Q and R respectively. If PA = 4 cm, BP = 3 cm and AC = 11cm, then length of BC is



A. 11 cm

B. 10 cm

C. 14 cm

D. 15 cm

Answer

Given:

PA = 4 cm

BP = 3 cm

AC = 11cm

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.





By above property,

AP = AR = 4 cm (tangent from A)

```
BP = BQ = 3 \text{ cm} \text{ (tangent from B)}
```

```
QC = RC (tangent from C)
```

Clearly,

RC = AC - AR

 \Rightarrow RC = 11 cm - 4 cm

 \Rightarrow RC = 7 cm

Now,

```
BC = BQ + QC
```

```
\Rightarrow BC = BQ + RC [: QC = RC]
```

 \Rightarrow BC = 3 cm + 7 cm

 \Rightarrow BC = 10 cm

Hence, BC = 10 cm

36. Question

In Fig. 10.98, a circle touches the side DF of AEDF at H and touches ED and EF produced at K and M respectively. If EK = 9 cm, then the perimeter of Δ EDF is



A. 18 cm

B. 13.5 cm

C. 12 cm

D. 9 cm

Answer

Given:

EK = 9 cm





<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

```
By above property,

EM = EK = 9 \text{ cm} (tangent from E)

DK = DH (tangent from D)

FM = FH (tangent from F)

Now,

Perimeter of \Delta EDF = ED + DF + FE

\Rightarrow Perimeter of \Delta EDF = (EK - KD) + (DH + HF) + (EM - MF)

[\because ED = EK - KD

DF = DH + HF

FE = EM - MF]

\Rightarrow Perimeter of \Delta EDF = EK - KD + DH + HF + EM - MF

\Rightarrow Perimeter of \Delta EDF = EK - DH + DH + HF + EM - HF [<math>\because DK = DH and FM = FH]

\Rightarrow Perimeter of \Delta EDF = EK + EM

\Rightarrow Perimeter of \Delta EDF = 9 \text{ cm} + 9 \text{ cm}

\Rightarrow Perimeter of \Delta EDF = 18 \text{ cm}
```

Hence, Perimeter of Δ EDF = 18 cm

37. Question

In Fig, DE and DF are tangents from an external point D to a circle with centre A. If DE = 5 cm and DE \perp DF, then the radius of the circle is







- A. 3 cm
- B. 5 cm
- C. 4 cm
- D. 6 cm

Answer

Given:

- DE = 5 cm
- DE⊥ DF







Join AE and AF

<u>Property 1:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

<u>Property 2:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 3:</u> Sum of all angles of a quadrilateral = 360°.

By property 1,

EF = ED = 5 cm(tangent from E)

And,

AE = AF [radius]

By property 2, $\angle AED = 90^{\circ}$ and $\angle AFD = 90^{\circ}$.

Also,

 $\angle EDF = 90^{\circ} [\because ED \bot EF]$

By property 3,

 $\angle AED + \angle AFD + \angle EDF + \angle EAF = 360^{\circ}$

 \Rightarrow 90° + 90° + 90° + \angle EAF = 360°

 $\Rightarrow \angle EAF = 360^{\circ} - (90^{\circ} + 90^{\circ} + 90^{\circ})$

⇒ ∠EAF = 360° - 270°

⇒ ∠EAF = 90°

 \therefore All angles are equal and adjacent sides are equal \therefore AEDF is a square.

Hence, all sides are equal

 \Rightarrow AE = AF = ED = EF = 5 cm

Hence, Radius of circle = 5 cm

38. Question

In Fig. 10.100, a circle with centre O is inscribed in a quadrilateral ABCD such that, it touches sides BC, AB, AD and CD at points P, Q, R and S respectively. If AB = 29 cm, AD = 23 cm, $\angle B$ = 90° and DS = 5 cm, then the radius of the circle (in cm) is







D. 15

Answer

Given:

AB = 29 cm

AD = 23 cm

∠B = 90°

DS = 5 cm

<u>Property 1:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

<u>Property 2:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 3:</u> Sum of all angles of a quadrilateral = 360°.

By property 1,

BP = BQ (tangent from B)

DS = DR = 5 cm (tangent from D)

```
AR = AQ (tangent from A)
```

Also,

OQ = OP (radius)

By property 2, $\triangle OQB$ is right-angled at $\angle OQB$ (i.e., $\angle OQB = 90^{\circ}$) and $\triangle OPB$ is right-angled at $\angle OPB$ (i.e., $\angle OPB = 90^{\circ}$).

Now by property 3,

 $\angle PBC + \angle BQO + \angle QOP + \angle OPB = 360^{\circ}$





 \Rightarrow 90° + 90° + \angle QOP + 90° = 360° $\Rightarrow 270^{\circ} + \angle QOP = 360^{\circ}$ ⇒ ∠QOP = 360° - 270° $\Rightarrow \angle QOP = 90^{\circ}$: adjacent sides (i.e., BP = BQ and OQ = OP) are equal and all angles are 90° ∴ quadrilateral OPBQ is a square Now, AD = 23 cm \Rightarrow AR + RD = 23 cm [: AD = AR + RD] \Rightarrow AR + 5 cm = 23 cm \Rightarrow AR = 23 cm - 5 cm \Rightarrow AR = 18 cm \Rightarrow AQ = AR = 18 cm Now, AB = 29 cm \Rightarrow AQ + QB = 29 cm [: AD = AR + RD] \Rightarrow 18 cm + QB = 29 cm \Rightarrow QB = 29 cm - 18 cm \Rightarrow QB = 11 cm ·· OPBQ is a square

∴ OP = BQ = 11 cm

Hence, radius = 11 cm

39. Question

In a right triangle ABC, right angled at B, BC = 12 cm and AB = 5 cm. The radius of the circle inscribed in the triangle (in cm) is

A. 4

- B. 3
- C. 2
- D. 1

Answer

Given:





<u>Property 1:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

<u>Property 2:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 3:</u> Sum of all angles of a quadrilateral = 360°.

By property 1,

AP = AQ (Tangent from A)

BP = BR (Tangent from B)

CR = CQ (Tangent from C)

 \because ABC is a right-angled triangle, \because by Pythagoras Theorem

 $AC^2 = AB^2 + BC^2$

 $\Rightarrow AC^2 = 5^2 + 12^2$

 $\Rightarrow AC^2 = 25 + 144$

 $\Rightarrow AC^2 = 169$

 $\Rightarrow AC = \sqrt{169}$

 \Rightarrow AC = 13 cm

Clearly,

AQ + QC = AC = 13 cm

 \Rightarrow AP + RC = 13 cm [: AQ = AP and QC = RC]





Also,

AB + BC = 5 cm + 12 cm = 17 cm \Rightarrow AP + PB + BR + RC = 17 cm [: AB = AP + PB and BC = BR + RC] \Rightarrow AP + RC + PB + BR = 17 cm \Rightarrow 13 cm + BR + BR = 17 cm [: AP + RC = 10 cm and PB = BR] \Rightarrow 13 cm + 2BR = 17 cm \Rightarrow 2BR = 17 cm - 13 cm = 4 cm \Rightarrow BR = $\frac{4}{2}$ cm \Rightarrow BR = 2 cm Now, $\angle BPO = 90^{\circ} [By property 2]$ $\angle BRO = 90^{\circ}$ [By property 2] $\angle PBM = 90^{\circ}$ [Given] Now by property 3, $\angle BPO + \angle BRO + \angle PBM + \angle ROP = 360^{\circ}$ $\Rightarrow \angle ROP = 360^{\circ} - (\angle BPO + \angle BRO + \angle PBM)$ $\Rightarrow \angle ROP = 360^{\circ} - (90^{\circ} + 90^{\circ} + 90^{\circ})$ ⇒ ∠ROP = 360° - 270° $\Rightarrow \angle ROP = 90^{\circ}$ Now, $\therefore \angle ROP = 90^{\circ}$ and BP = BR which are adjacent sides ∴ Quadrilateral PBRO is a square

 \Rightarrow PO = BR = 2 cm

Hence, Radius = 2 cm

40. Question

Two circles touch each other externally at P. AB is a common tangent to the circle touching them at A and B. The value of \angle APB is

A. 30°

B. 45°

C. 60°

D. 90°





Answer



Draw a tangent from a point T on B to P.

<u>Property 1:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

<u>Property 2:</u> Sum of all angles of a triangle = 180°.

```
By property 1,
TA = TP (tangent from T)
TB = TP (tangent from T)
Now in \triangle ATP,
TA = TP
\therefore \angle APT = \angle PAT
And in \Delta BTP,
TB = TP
\therefore \angle BPT = \angle PBT
By property 2,
\angle APB + \angle PBA + \angle PAB = 180^{\circ}
\Rightarrow \angle APB + \angle PBT + \angle PAT = 180^{\circ}
\Rightarrow \angle APB + \angle BPT + \angle APT = 180^{\circ} [:: \angle APT = \angle PAT \text{ and } \angle BPT = \angle PBT]
\Rightarrow \angle APB + \angle APB = 180^{\circ} [\because \angle APB = \angle BPT + \angle APT]
\Rightarrow 2 \angle APB = 180^{\circ}
\Rightarrow \angle APB = \frac{180^{\circ}}{2}
\Rightarrow \angle APB = 90^{\circ}
```

Hence, $\angle APB = 90^{\circ}$

41. Question

In Fig. 10.101, PQ and PR are two tangents to a circle with centre O. If \angle QPR = 46°, then \angle QOR equals



- A. 67°
- B. 134°
- C. 44°
- D. 46°

Answer

Given:

 $_{\angle}$ QPR = 46°

<u>Property 1:</u> The tangent at a point on a circle is at right angles to the radius obtained by joining center and the point of tangency.

<u>Property 2:</u> Sum of all angles of a quadrilateral = 360°.

By property 1, $\triangle OQP$ is right-angled at $\angle OQP$ (i.e., $\angle OQP = 90^{\circ}$) and $\triangle ORP$ is right-angled at $\angle ORP$ (i.e., $\angle ORP = 90^{\circ}$).

Now by property 2,

 $\angle OQP + \angle ORP + \angle QOR + \angle QPR = 360^{\circ}$

 $\Rightarrow \angle QOR = 360^{\circ} - (\angle OQP + \angle ORP + \angle QPR)$

 $\Rightarrow \angle \mathsf{ROP} = 360^\circ - (90^\circ + 90^\circ + 46^\circ)$

- ⇒ ∠ROP = 360° 226°
- $\Rightarrow \angle ROP = 134^{\circ}$

Hence, $\angle ROP = 134^{\circ}$

42. Question

In Fig. 10.102, QR is a common tangent to the given circles touching externally at the point T. The tangent at T meets QR at P. If PT = 3.8 cm, then the length of QR (in cm) is

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A. 3.8

B. 7.6

C. 5.7

D. 1.9

Answer

Given:

PT = 3.8 cm

<u>Property 1:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By property 1,

PQ = PT (Tangent from P)

```
PR = PT (Tangent from P)
```

Now,

QR = PQ + PR

 \Rightarrow QR = PT + PT

 \Rightarrow QR = 3.8 cm + 3.8 cm

⇒ QR = 7.6 cm

Hence, QR = 7.6 cm

43. Question

In Fig. 10.103, a quadrilateral ABCD is drawn to circumscribe a circle such that its sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively. If AB = x cm, BC = 7 cm, CR = 3 cm and AS = 5 cm, then x =







- B. 9
- C. 8
- D. 7

Answer

Given:

AB = x cm

BC = 7 cm

CR = 3 cm

AS =5 cm

<u>Property:</u> If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

- AP = AS (tangent from A)
- BP = BQ (tangent from B)
- CR = CQ (tangent from C)

```
DR = DS (tangent from D)
```

Clearly,

```
QB = CB - CQ
```

```
\Rightarrow QB = CB - CR [:: CQ = CR]
```

```
\Rightarrow QB = 7 cm - 3 cm
```

 \Rightarrow QB = 4 cm

Now,

AB = AP + PB

 $\Rightarrow AB = AS + QB$



 $\Rightarrow AB = 5 \text{ cm} + 4 \text{ cm}$

 $\Rightarrow AB = 9 \text{ cm}$

 $\Rightarrow AB = x = 9 \text{ cm}$

Hence, x = 9 cm



